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TECHNIQUES FOR THE IMPROVEMENT OF
ASTRONOMIC POSITIONING IN THE FIELD

BY

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The paper deals with new methods and techniques for improving astronomic positioning in the field. Latitude and longitude are obtained by observing transit times of pairs of stars over a fixed vertical plane, independent of azimuth and zenith distances. A unique solution is derived for each pair. Therefore, short periods of clear skies can be utilized. The time and therefore cost to ertablish one station is accordingly reduced. Higher accuracy in latitude can be obtained by observing transit times of star pairs over the prime vertical, where the parallactic angle reaches its maximum value. The vertical plane of observation

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can be fixed within 90 arc seconds with respect to the prime vertical without changes in the star's parallactic angle, and a function of it, the latitude, can the be computed. The star transit times over different vertical lines are thereby reduced to the central line or collination plane, as a function of the parallactic angle. Higher accuracy in longitude can be achieved by observing the transit times of pairs of stars over a vertical plane fixed within 20 arc minutes with respect to the meridian plane. Each individual star pair will determine a solution. Since each pair does not depend on azimuth orientation, the star pairs can be chosen arbitrarily with respect to declination or zenith distance, and short periods of clear sky observations can be utilized. When several pairs are observed an adjustment can be carried out through the equations of condition that allow one to detect errors in either the transit times or in the star's right ascensions.

ABSTRACT

The paper deals with new methods and techniques for improving astronomic positioning in the field. Latitude and longitude are obtained by observing transit times of pairs of stars over a fixed vertical plane, independent of azimuth and zenith distances. A unique solution is derived for each pair. Therefore, short periods of clear skies can be utilized. The time and therefore cost to establish one station is accordingly reduced.

Higher accuracy in latitude can be obtained by observing transit times of star pairs over the prime vertical, where the parallactic angle reaches its maximum value. The vertical plane of observation can be fixed within 90 arc seconds with respect to the prime vertical without changes in the star's parallactic angle, and a function of it, the latitude, can then be computed. The star transit times over different vertical lines are thereby reduced to the central line or collimation plane, as a function of the parallactic angle.

Higher accuracy in longitude can be achieved by observing the transit times of pairs of stars over a vertical plane fixed within 20 arc minutes with respect to the meridian plane. Each individual star pair will determine a solution. Since each pair does not depend on azimuth orientation, the star pairs can be chosen arbitrarily with respect to declination or zenith distance, and short periods of clear sky observations can be utilized. When several pairs are observed an adjustment can be carried out through the equations of condition that allow one to detect errors in either the transit times or in the star's right ascensions.

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INTRODUCTION

Accurate time and longitude are determined by the astronomers by observing transits of stars over the meridian. The chief instrument used is a transit instrument which must be placed close to the meridian plane within a few seconds of arc. A suitable number of observing stars is required. Usually ten or twelve stars are to be observed.

There are two general methods of observing stars for a time determination. In one, half the set of stars are observed with the axis of the telescope fixed in one position (clamp west or east) and the other half in a reverse telescope position (clamp east or west). In each half set, one star must be situated near the pole. In the other method, each star is observed before crossing the meridian's plane with the axis of the telescope in clamp west, and after crossing the meridian with the axis of the telescope reversed.

The first method involves the computation of four unknowns. The second method involves two unknowns. Half of the suitable stars to be used for a time determination must be a north of the zenith, the other half at south of the zenith, and in such position that their azimuth errors balance each other. It takes one hour or more to make a complete observation of the set.

In making the computation, it is asssumed there is not rotation of the pier during the period of time required by observing program, so the line of collimation remain constant with respect to the meridian plane.

Most textbooks do not give formulas for solving the longitude problem other than the well-known Mayer equation. Many authors have also realized that the Mayer equation leads to erroneous results when a larger number of stars are used in a method of least squares solution. Black [1951] was the first to

apply the function of zenith distances as coefficients in his new azimuth equation. Dufour [1958] approached the problem from a slightly different direction and concluded that examination of the nature of the errors requires some changes in the universally accepted Mayer formula.

The advantages of our method with respect to the classical method can be set up as follows:

1. In the new method observation of only one pair of stars can give an accuracy that matches the accuracy that can be obtained by the classical method of observing ten stars.
2. It is independent of the instrumental azimuth error.
3. The instrument can be placed in any vertical plane, within an error azimuth of 20 minutes of arc with respect to the meridian plane.
4. Does not require selection of the stars in order for their azimuth coefficients to be balanced between the north and south stars, as is required in the classical method.
5. The stars forming a pair are arbitrarily chosen with respect to the star's declination.
6. Because the elapse of time, between the two stars of a pair is very small, the clock correction is independent of the rotation of the pier.
7. Our method reduces the number of unknowns that appear in the classical methods by 50%.
8. Any star can be combined with another forming a pair that gives an insight not only about the accuracy obtainable but can be pointed out if an error in the star's position exists.

The procedure of determining the latitude of a station comprises observations of pairs of stars, east and west of the observer, over the prime vertical or in a plane close to it. Independent of the clock correction, the latitude is based on the condition that those stars, whose declinations are less than the latitude, acquire maximum parallactic angle when they transit the prime vertical. The instrument remains fixed in azimuth while observations are made for several different stars. The telescope is pointed toward each star in such a way that its image moves across the central field of view and the chronometer times are recorder each time the star's image crosses one of the vertical wire with which the instrument is provided. The set of observed values for each star is reduced to obtain the value corresponding to the central wire or collimation plane and used to obtain the latitude. Based on Taylor's theorem, the formula for reducing the observations on the side wires to the middle wire is given. Errors in the transit times can be adjusted under the condition that the stars' zenith distance rate with respect to time is constant.

E. Buschmann (1) analyzed about 3,000 time observations at the Geodetic Institute of Potsdam. These observations were made by 9 observers with 4 transit instrument and one Danjon astrolabe. He concludes that metheoro-logical errors are unimportant while instrument errors predominate.

Based on Buschmann's investigation we introduce the concept of dynamical collimation error and the method of determining it, instead of using the static collimation error, as more precise and adequate for handling the effects of the observer on tracking the star's image motion.

Independent of the local station's coordinates, equations of condition were developed to detect errors either in the transit times or star's right ascension.

HIGH ACCURACY EVALUATION OF THE LATITUDE AS FUNCTION OF THE PARALLACTIC ANGLE OF STARS CROSSING THE PRIME VERTICAL

To this purpose pairs of stars are observed east and west of the meridian recording the transit times, over the prime vertical or near to it.
A theodolite is clamped in azimuth so the telescope is moving on the vertical plane.
The star parallactic angle has a relation to the azimuth through the equation

$$\cos \delta \sin P = \cos \phi \sin A \tag{1}$$

where the azimuth A, is measured from the south, clockwise.

Those stars, whose declinations are comprised between the equator and the observer's latitude, cross the prime vertical at two points: east when $A_E = 270°$ and west when $A_W = 90°$, with equal zenith distances. At these points the parallactic angle reaches its maximum value, as can be seen by differentiating equation (1), which gives

$$\frac{dP}{dA} = \frac{\cos \phi \cos A}{\cos \delta \cos P} \tag{2}$$

so in the prime vertical,

$$\left(\frac{dP}{dA}\right)_{A = 90°} = 0 \tag{3}$$

Therefore while the azimuth increases continuously with respect to time, the parallactic angle does not. It increases continuously, from zero to its maximum, at the time when the star is over the prime vertical, and then decreases again from that point. Consequently, the parallactic angle has equal values north and south with respect to the prime vertical.
We use this condition to determine latitude as function of the

parallactic angle.

EVALUATION OF LATITUDE

We have found that the latitude can be obtained through the equation

$$\tan \phi = \frac{\sin t}{\cos \delta \tan P} + \cos t \tan \delta \tag{4}$$

where the parallactic angle P, can be known from observations of pairs of stars, as can be seen later, but the hour angle t, is unknown. The influence of an error dt in the hour angle t, produces an error $d\phi$, upon the latitude as follows,

$$(1 + \tan^2\phi)d\phi = \left(\frac{\cos t}{\cos \delta \tan P} - \sin t \tan \delta\right)dt \tag{5}$$

From equation

$$\cos A = \cos t \cos P - \sin t \sin P \sin \delta \tag{6}$$

and equation (1) we obtain

$$d\phi = \frac{\cos^2\phi \cos A}{\cos \delta \sin P} dt = \frac{\cos \phi}{\tan A} dt \tag{7}$$

Our observations are made very close to the prime vertical, in a plane oriented in an azimuth $A = 90° \pm \epsilon$.
Considering a gross error $\epsilon = 3'$, we obtain

$$d\phi < 0.0009 \cos \phi dt \tag{8}$$

Therefore a rough approximation of the hour angle must be known. To evaluate the hour angle we proceed as follows.

Let θ_v the star's transit times over the prime vertical, and θ be the transit time over the vertical plane, whose azimuth is $90° + \epsilon$. The interval of time $(\epsilon_v - \theta)$ is obtained from the relation

$$\delta t = \left(\theta_v - \theta\right) \frac{dt}{dA} \tag{9}$$

and the changes in the parallactic angle is

$$\delta P = \frac{1}{2} \frac{d^2P}{dt^2} (\theta_v - \theta)^2 \tag{10}$$

From equation (1) we obtain

$$\frac{d^2P}{dt^2} = - \tan P \sin^2 \phi \tag{11}$$

In reference (1) the author has found that the rates of changes in the star's hour angle and parallactic angle, are related to the general equation

$$\sin \phi \frac{dt}{dA} = 1 - \frac{dP}{dA} \cos Z \tag{12}$$

For observations in the prime vertical $\frac{dP}{dt} = 0$ (equation 3), so it follows that

$$\left(\frac{dt}{dA}\right)_{A=90} = \operatorname{cosec} \phi \tag{13}$$

so equation (9) gives for $\epsilon = 180''$,

$$\theta_v - \theta = 180'' \operatorname{cosec} \phi = 12^s \operatorname{cosec} \phi \tag{14}$$

then the changes in the parallactic angle between the value in the prime vertical and the value on the vertical plane close to an error of 3' with respect to the first is

$$\delta P = \tfrac{1}{2}\,\varepsilon^2 \tan P \tag{15}$$

Expressing δP in seconds of arc we get

$$\delta P = -\tfrac{1}{2}\left(\frac{3}{60}\right)^2 \frac{\pi}{180} \times 3600'' \tan P \tag{16}$$

$$\delta P = -\,0''.079 \tan P \tag{17}$$

Using star's declination $\delta = \phi - 5°$, dP never goes over half second of arc, therefore we can use in equation (4), with sufficient accuracy, the hour angle in the prime vertical.

The hour angle, when the star is over the prime vertical, can be derived as function of the parallactic angle, as follows

$$\tan t_v = \cot P \operatorname{cosec} \delta \tag{18}$$

Now in view of the low influence of an error dt upon the latitude, as it has been demonstrated through the equations (7) and (8), the hour angle t_v can be used as a good approximation of the angle t, in equation (4). Therefore the final equations for computing the latitude are

$$\tan \phi = \frac{\sin t}{\cos \delta \tan P} + \cos t \tan \delta \tag{19}$$

with

$$\tan t = \cot p \operatorname{cosec} \delta \tag{20}$$

Furthermore, if the observations are made in a plane within 90" of the prime vertical

$$A = 90° \pm 90''$$

the parallactic angle in this plane should be almost the same as its value over the prime vertical, and the latitude is defined from

$$\tan \phi = \frac{\sqrt{\cos^2 P + \tan^2 \delta}}{\sin P} \tag{21}$$

When several pairs of stars are observed an adjustment may be considered to increase accuracy in the parallactic angles, under the condition that

$$\cos \delta_i \sin P_i + \cos \delta_j \sin P_j = 0 \tag{22}$$

where the index i refers to the stars at east and j to the stars at west,

$$i = 1, 2, 3 \ldots$$

$$j = 1, 2, 3 \ldots$$

COMPUTATION OF THE PARALLACTIC ANGLE

The parallactic angle is obtained from observations of pairs of stars as function of transit times, east and west of the observer, over a fixed vertical plane oriented nearly to the prime vertical. The instrument is clamped in azimuth so the telescope is moving on the vertical plane. The axis of rotation is perpendicular to the vertical plane and lies almost in the intersection of the planes of the meridian and horizon.

Let figure z be zenith, B and C the west and east stars. Consider first that there are no instrumental errors. Let index "w" refer to the west star and "e" to the east star.

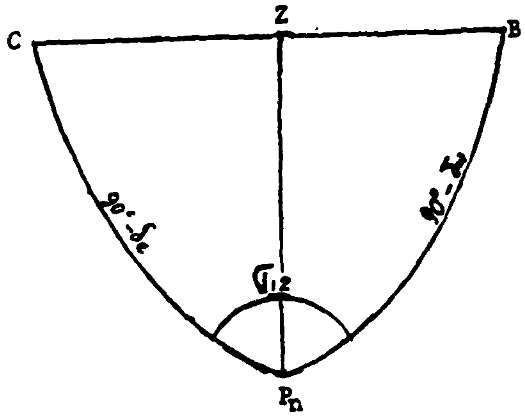

Figure 1 , showing the astronomical triangle

The angle at the pole, σ_{12}, has the value

$$\sigma_{12} = (\theta_w - \theta_E)(1+\mu) - (\alpha_w - \alpha_E) \qquad (23)$$

Where

θ_w, θ_E = star's transit time over the vertical central thread

μ = clock rate

Applying Napier's formula we have

$$\tan \tfrac{1}{2}(B + C) = \frac{\cos \frac{1}{2}(\delta_w - \delta_e)}{\sin \frac{1}{2}(\delta_w + \delta_e)} \cot \tfrac{1}{2}\sigma_{12} \qquad (24)$$

$$\tan \tfrac{1}{2}(B - C) = \frac{\sin \frac{1}{2}(\delta_w - \delta_e)}{\cos \frac{1}{2}(\delta_w + \delta_e)} \cot \tfrac{1}{2}\sigma_{12} \qquad (25)$$

Let

$$\frac{1}{2}(B + C) = x$$

$$\frac{1}{2}(B - C) = y \qquad (26)$$

The star's parallactic angles are:

$$P_w = x + y$$

$$P_e = 360° - x + y \qquad (27)$$

CORRECTION TO THE ANGLE σ_{12} DUE TO ERRORS OF COLLIMATION AND INCLINATION

The theodolite is said to be in the prime vertical when the telescope describes a great circle of arc coincident with the prime vertical. The rotation axis is then normal to the prime vertical plane, and lies in the intersection of the meridian and horizontal planes. But due to the fact that there exist collimation and inclination error, corrections must be introduced to the observation times. Consider first the collimation error.

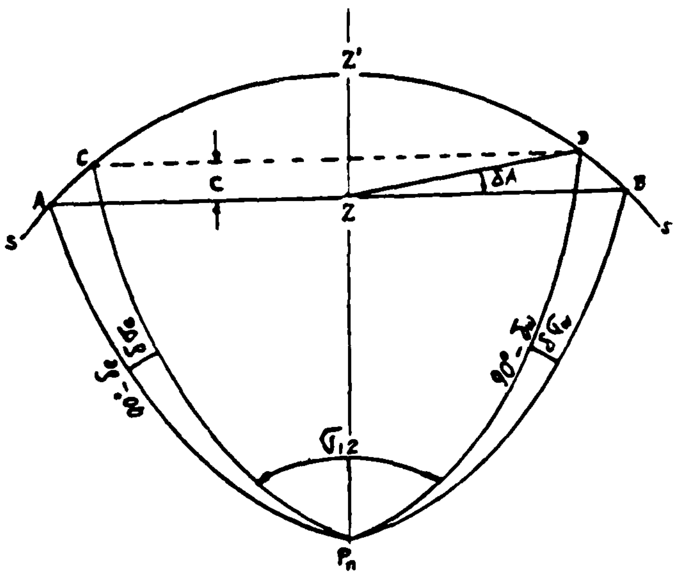

Figure 2, showing the astronomical triangle

Let ZP_n, Figure 3, be the meridian, AZB the prime vertical of the observer. ACDB the diurnal circle of a star which crosses the meridian between the zenith and the equator.

Such star crosses the prime vertical above the horizontal plane at two points A and B on opposite sides of the zenith and at equal zenith distances from the meridian. We observe the star's transit at these two points when the instrument is perfectly adjusted in the prime vertical. Due to the collimation error the telescope will not describe

the great circle of arc AZB but the smallest CZ^1D circle of arc. The star's transit times refer to the star position over A and B. Draw the great circle of declination through C and D. The angle $CP_nD < AP_nB$. Let

$$\delta\sigma_e = AP_nC$$

$$\delta\sigma_w = DP_nB$$

$$\bar{\sigma}_{12} = CP_nD$$

The true angle σ_{12} to be used in equations (24) and (25), is

$$\sigma_{12} = \bar{\sigma}_{12} + \delta\sigma_e + \delta\sigma_w$$

Let us determine the values of $\delta\sigma_e$ and $\delta\sigma_w$. D and B lie on the small circle DB (the parallel of declination) of which P_n is the pole. As the star moves, owing to the diurnal motion, from D to B, the angle $\bar{\sigma}_{12}$ increases in $\delta\sigma_w$ and its azimuth increases in δ_A. At west the star is observed at D before it crosses over the prime vertical at B. At east the star is observed at C, after it crosses the prime vertical over A.

The collimation error is always small. It is well known that the collimation affects the azimuth in the amount

$$\delta A_c = c \operatorname{cosec} Z$$

The σ angle variation with respect to azimuth, in the prime vertical,

is therefore

$$d\sigma/dA = \text{cosec}\ \phi$$

$$\delta\sigma_c = \frac{d\sigma}{dA}\,\delta A_c$$

so
$$\delta\sigma_c = c\ \text{cosec}\,\phi\ \ \text{cosec}\ Z \qquad (29)$$

The effect of inclination error affects the azimuth in the amount

$$\delta A_i = i\ \cot\ z$$

and the correction to the σ_{12} angle is

$$\delta\sigma_i = i\ \cot\ z\ \text{cosec}\ \phi \qquad (30)$$

The total correction due to collimation and inclination error is for a star in the east

$$d\sigma_e = \frac{C + i_e \cos Z_e}{\sin\phi\ \sin Z_e} \qquad (31)$$

and

$$d\sigma_w \quad \frac{C + i_w \cos Z_w}{\sin\phi\ \sin Z_w} \qquad (32)$$

for a star at west. Therefore the angle σ_{12} is

$$\sigma_{12} = (\Theta_w - \Theta_e)(1 + \mu) - (\alpha_w - \alpha_e) + d\sigma_w + d\sigma_e \qquad (33)$$

OBSERVATION OF ONE STAR

A given star may be observed on the fixed vertical plane, east and west of the observer. Consider that this plane is close to the prime vertical within 70". The angle at the pole σ_{12} is obtained from

$$\sigma_{12} = (\Theta_w - \Theta_e)(1 + n) - (\alpha_w - \alpha_e) + \frac{2c+(i_e + i_w)\cos Z}{\sin \phi \sin Z} \tag{34}$$

the parallactic angle P, from

$$\tan P = \operatorname{cosec} \delta \cot \tfrac{1}{2} \sigma_{12} \tag{35}$$

and the latitude from

$$\tan \phi = \frac{\sqrt{\cos^2 P + \tan^2 \delta}}{\sin P} \tag{36}$$

If the latitude of the observer is completely unknown, then an approximate value of the parallactic angle P, can be computed by disregarding the collimation and inclination error. With this approximate value the zenith distance can be computed from

$$\cos Z = \frac{\tan \delta}{\sqrt{\cos^2 P + \tan^2 \delta}} \tag{37}$$

and ϕ^1 from (36), so the last right term in equation (34), can be obtained and the right evaluation of P and ϕ can be carried out.

As an example, we give the results obtained from observations that were taken by Hansen in Helgoland with transit instrument in the prime vertical, Astron. Nach, Vol. VI, p. 117.

Observation of August 3, 1824, is considered.

δ	$51° \ 10' \ 58''.04$
η	$+ \ 4^s.27$ per day
$1 + \eta$	1.0000494
Θ_w	$19^h \ 29^m \ 44^s.81$
Θ_e	16 11 27.61
$\Theta_w - \Theta_e$	3 18 17.20
$\frac{1}{2}\sigma'_{12} = \frac{15}{2}(\Theta_w - \Theta_e)(1 + \eta)$	$24° \ 47' \ 13''.50$

Preliminary parallactic angle: . . . . $\tan P' = \operatorname{cosec}\delta \tan \frac{1}{2}\sigma'_{12}$

Zenith distance: . . . . $\cos Z = \dfrac{\tan\delta}{\sqrt{\cos^2 P + \tan^2\delta}}$

P.		$70° \ 07' \ 33''.46$
Z.		$18° \ 07' \ 23''.50$
Collimation	c =	$-2''19$
Inclination	i =	$-1''.18$
$\frac{1}{2}(d\Theta_i + d\Theta_c) = \dfrac{c + i\cos Z}{\cos\delta\cos P}$		$-19''02$
$\frac{1}{2}\sigma_{12} = \frac{1}{2}\sigma'_{12} + \frac{1}{2}(d\Theta_i + d\Theta_c)$		$24° \ 46' \ 54''.48$

$$\tan P = \operatorname{cosec}\delta \cot\frac{1}{2}\sigma_{12}$$

P.	$70° \ 07' \ 49''44$

$$\tan\phi = \frac{\sqrt{\cos^2 P + \tan^2\delta}}{\sin P}$$

ϕ.	$54° \ 10' \ 46''.07$

This value of ϕ is obtained independently of the clock correction and has a difference of 0.01" with respect to the Hansen value.

REDUCTION OF OBSERVATIONS TO CENTRAL TIME

The stars' transit times considered in equations (23) and (33) are to be taken with respect to the central reticule line, but the stars actually are observed over different vertical lines. The times are to be reduced to the central or middle line. We proceed, therefore, to investigate the formula for reducing the observations on the side lines or collimation plane.

Let

$A_i - A_o$ = the horizontal interval of any given line with respect to the middle line.

T_i = the transit time over the side line.

$i = 1, 2, 3, \ldots n$

According to Taylor's theorem the star's transit time over any vertical line is reduced to the central line, or collimation plane, through the equation

$$T_o = T_i + \frac{dt}{dA}(A_o - A_i) + \frac{1}{2}\frac{d^2T}{dA^2}(A_o - A_i)^2 + \frac{1}{6}\frac{d^3T}{dA^3}(A_o - A_i)^3 \qquad (38)$$

The Taylor's coefficient, for observations over the prime vertical, were derived as follows:

$$\left(\frac{dt}{dA}\right)_{A=\pm 90} = \operatorname{cosec}\phi \qquad (39)$$

$$\left(\frac{dt}{dA}\right)_{A = \mp 90°} = \frac{\sin z}{\cos \delta \cos P} \tag{39}$$

$$\left(\frac{d^2t}{dA^2}\right)_{A = \pm 90°} = \frac{\cos z \sin z \tan P}{\cos \delta \cos P} \tag{40}$$

$$\left(\frac{d^3t}{dA^3}\right)_{A = \pm 90°} = -2 \frac{\sin^3 z \tan^2 P}{\cos \delta \cos P} \tag{41}$$

The parallactic angle P is evaluated as function of the stars' pair transit times over the central wire through the equations (23) to (27). As function of P, we obtain the star zenith distance from

$$\tan z = \cos P \ \cot \delta$$

As can be seen, the Taylor's coefficients can be obtained independent of the station astronomic coordinates. The evaluation of these coefficients are shown in Appendix B.

DETERMINATION OF LONGITUDE OR CLOCK CORRECTION

The author has shown that if two stars are observed on their transit over a fixed vertical plane, north and south of the observer, a relation between the star's coordinates and their transit times is as follows

$$\tan \delta_s \sin t_n - \tan \delta_n \sin t_s = \tan \phi \sin (t_s - t_n) \tag{42}$$

where index s, n, refer to south and north, and

$$\begin{aligned} t_s &= \theta_s - \alpha_s + \Delta T \\ t_n &= \theta_n - \alpha_n + \Delta T \end{aligned} \tag{43}$$

ϕ = observer's latitude

ΔT = clock correction

θ_s, θ_n = star's transit times

Equation (42) allows one to compute the observer's latitude if the clock correction or longitude is known. On the other hand if the latitude is known then the clock correction or longitude can be computed. Only transit times of pairs of stars are used. A unique solution is derived per pair. Therefore, short periods of clear skies can be utilized. The time and therefore cost to establish one station is accordingly reduced. When bath latitude and longitude are to be determined, then one star pair must be observed in a vertical plane fixed in an azimuth within 10-30 degrees, and second pair in another plane fixed between 330-350 degrees in azimuth. The reason for observing two pairs on different planes is due to the fact that observations of pairs on the same plane linear dependent. A combined astronomic longitude-latitude determination shall be developed in a sequence report.

Developing $\sin (t_n - t_s)$, equation (42) can be arranged as follows

$$\sin t_n(\tan \delta_s - \tan\phi \cos t_s) - \sin t_s (\tan \delta_n - \tan \phi \cos t_n) = 0 \tag{44}$$

Lets observe the stars in a vertical plane within a few arc minutes of the meridian. The hour angles t_s and t_n are small and opposed in signs, so we can then consider that:

$$\sin t_s = t_s - \frac{t_s^3}{6} \qquad \sin t_n = t_n - \frac{t_n^3}{6}$$
$$\cos t_s = 1 - \frac{t_s^2}{2} \qquad \cos t_n = 1 - \frac{t_n^2}{2} \tag{45}$$

So equation (44) can be rewritten as follows

$$t_n(\tan\delta_s - \tan\phi) - t_s(\tan\delta_n - \tan\phi)$$
$$- \frac{t_n^3}{6}(\tan\delta_s - \tan\phi) + t_s^3(\tan\delta_n - \tan\phi) = 0 \tag{46}$$

The influence of the last two terms can be analyzed and evaluated as follows: From the condition that in the meridian is

$$\left(\frac{d^2A}{dt^2}\right)_{A=0} = 0$$

and

$$\left(\frac{dA}{dt}\right)_{A=0} = \frac{\sin z}{\cos\delta}$$

we have

$$t_s = \frac{\sin z_s}{\cos\delta_s}\overline{\delta A} \qquad t_n = \frac{\sin z_n}{\cos\delta_s}\overline{\delta A} \tag{47}$$

Let two parameters, m and n, be related as follows:

$$m = \frac{1}{2}(t_n^3 + t_s^3) \qquad n = \frac{1}{2}(t_n^3 + t_s^3)$$

from which the last two terms in equation (45) are equal to

$$m(\tan\delta_n - \tan\delta_s) + n(2\tan\phi - \tan\delta_s - \tan\delta_n) \tag{49}$$

From equations (47) we found

$$m = \left(\left(\frac{\sin z_s}{\cos \delta_s}\right)^3 + \left(\frac{\sin z_n}{\cos \delta_n}\right)^3 \right) \frac{\overline{\delta A^3}}{2}$$

$$n = \left(\left(\frac{\sin z_n}{\cos \delta_n}\right)^3 - \left(\frac{\sin z_s}{\cos \delta_s}\right)^3 \right) \frac{\overline{\delta A^3}}{2} \qquad (50)$$

in which

z_s, z_n = star's meridian zenith distance

δA = azimuth from south clockwise

Keeping $\delta A < 20'$ we have

$$\frac{\delta A^3}{2} < \frac{1}{2}\left(\frac{1}{3}\right)^3 \left(\frac{\pi}{180}\right)^2 < 0^s\!.0014 \qquad (51)$$

Consequently we can disregard the influence of the last two terms in equation (45), so we have the final equation valid for $\delta A < 20'$:

$$t_n(\tan \delta_s - \tan \phi) - t_s(\tan \delta_n - \tan \phi) = 0 \qquad (52)$$

Let

$$\Theta_s + \eta\,(\Theta_s - \Theta_o) - \alpha = \beta_s$$

$$\Theta_n + \eta\,(\Theta_n - \Theta_o) - \alpha = \beta_n \qquad (53)$$

where η, is the clock rate. Then

$$t_s = \beta_s + \Delta T$$

$$t_n = \beta_n + \Delta T \qquad (54)$$

Introducing the parameters

$$M_1 = \tan \delta_s - \tan \phi$$

$$M_2 = \tan \delta_n - \tan \phi \qquad (55)$$

the clock correction is computed from

$$\Delta T = \frac{\beta_n M_1 - \beta_s M_2}{M_2 - M_1} \tag{56}$$

Each pair of stars gives the clock correction independently one to each other.

If longitude is required, instead of the clock correction, then for the hour angles we must use

$$\begin{aligned} t_s &= (\Theta_g - \alpha)_s - \lambda_o - d\lambda \\ t_n &= (\Theta_g - \alpha)_n - \lambda_o - d\lambda \end{aligned} \tag{57}$$

where

λ_o = approximate value of longitude taken positive to the west

Θ_g = greenwich sidereal time

$d\lambda$ = correction to be computed

The correction $d\lambda$ is computed from

$$d\lambda = \frac{\beta_s M_2 - \beta_n M_1}{M_2 - M_1} \tag{58}$$

and the longitude λ, from

$$\lambda = \lambda_o + d\lambda \tag{59}$$

CORRECTIONS DUE TO COLLIMATION AND INCLINATION ERROR

Equation (56) was derived assuming no instrumental error existed. But due to the fact collimation and inclination errors exist, corrections must be introduced to the observed transit times. To deal with these corrections we consider that the collimation affects the azimuth in the amount

$$\delta A_c = c \operatorname{cosec} z$$

and the inclination in the amount

$$\delta A_i = i \cot z$$

and the combined effects to the azimuth, is

$$\delta A_c + \delta A_i = \frac{c + i \cos z}{\sin z} \tag{60}$$

The correction to the transit time over the vertical plane, located at an azimuth δA, with respect to the meridian plane, is

$$d\theta = \frac{c + i \cos z}{\sin z} \left(\frac{dt}{dA}\right)_{A = \delta A} \tag{61}$$

where

$$\left(\frac{dt}{dA}\right)_{A = \delta A} = \frac{\sin^2 z}{\sin \phi - \cos z \sin \delta} \tag{62}$$

so the correction to the transit time is

$$d\theta = \frac{(c + i \cos z) \sin z}{\sin \phi - \cos z \sin \delta} \tag{63}$$

which is the correction to be added or substrated, according to its sign, to the times Θ_s, Θ_n, in equation (53).

Consider the difference between the zenith distance Z_m, in the meridian and the zenith distance z_v, in the vertical plane of observations.

Let τ, be the elapsed time that the star takes to pass from the meridian to the plane of observation. According to Taylor's theorem, and since in the meridian is

$$\frac{dz}{dt} = \frac{d^3z}{dt^3} = 0$$

we have

$$z_v - z_m = \frac{1}{2} \left(\frac{d^2z}{dt^2}\right)_{A=0} \tau^2 \tag{64}$$

From the equation

$$\frac{dz}{dt} = \cos\phi \sin A$$

we get

$$\left(\frac{d^2z}{dt^2}\right)_{A=0} = \cos\phi \left(\frac{dA}{dt}\right)_{A=0} \tag{65}$$

and from

$$\left(\frac{dA}{dt}\right)_{A=0} = \frac{\cos\delta}{\sin z_m}$$

equation (64), becomes

$$z_v - z_m = \frac{1}{2} \frac{\cos\delta \cos\phi}{\sin z_m} \tau^2 \tag{66}$$

Expressing τ in degrees and $(z_v - z_m)$ in seconds of arc, we obtain

$$(z_v - z_m)'' = \frac{1}{2} \; \frac{\pi}{180} \; 3600'' \; \frac{\cos\phi \cos\delta}{\sin z} (\tau^\circ)^2 \tag{67}$$

which becomes

$$(z_v - z_m)'' = 31''4 \; \frac{\cos\phi \cos\delta}{\sin z} (\tau^\circ)^2 \tag{68}$$

Keeping $\delta A < 20'$, the interval of time is less than 4^m, and therefore

$$z_v - z_m < 60''$$

This fact, and because of the smallness of c and i, we may consider that

$$\left(\frac{dt}{dA}\right)_{A = \delta A} \approx \left(\frac{dt}{dA}\right)_{A = 0} = \frac{\sin z_m}{\cos\delta} \tag{69}$$

This allows the reduction of equation (61) to

$$d\theta = \frac{c + i \cos z_m}{\cos\delta} \tag{70}$$

Therefore for observations in a vertical plane within $\delta A = 20'$, with respect to the meridian plane, we always use equation (62) for the corrections due to collimation and inclination error.

The collimation error disappears when the star is observed before and after crossing the vertical plane. If the collimation is known the times Θ_s and Θ_n must be corrected with the corresponding values $d\Theta$ given by equation (70). But when the collimation is unknown stars that are observed with the instrument in direct position (clamp west or east) and other stars in the reverse position (clamp east or west), then the collimation error must be derived together with the clock correction.

In this case the equation to be used is

$$\Delta T \pm c \cdot C = \frac{\beta_s M_2 - \beta_n M_1}{M_2 - M_1} \quad \begin{array}{l} + \text{ direct} \\ - \text{ reverse} \end{array} \tag{71}$$

where M_1 and M_2 have the values shown in equation (55) and C has the value given by

$$c = \frac{\sin (\phi - \delta_s) - \sin (\phi - \delta_n)}{\cos \phi \sin (\delta_s - \delta_n)} \tag{72}$$

This process of computation must be used when observations are made with a theodolite, as the Wild T-4. We must bear in mind that a star cannot be tracked before and after crossing the vertical plane of observation and use the mean time of both observations as transit time.

It is not advisable to track the stars by inverting the theodolite, in the manner in which the "Broken-Telescope" Transit is used, unless the instrument is provided with a device to turn it quickly and accurately from the direct to the reversed position.

Let us apply this method to the reduction of a simulated observations in a latitude $\phi = 40°$, assuming an error of 20 of arc in azimuth. The influence of an error $\delta\phi$ in the latitude ϕ, produces an error $\delta(\Delta T)$, upon the clock correction as follows:

$$\delta(\Delta T) = \frac{(\beta_s - \beta_n)\ (1 + \tan^2\phi)\ \sin 1''}{\tan \delta_n - \tan \delta_s} \quad \delta\phi$$

The reduction of the observations are shown on Table I . The clock correction is computed according to equation (56).

TABLE 1

Declination		Zenith Distance	Transit Time Θ	Right Ascension α	$\beta = \Theta - \alpha$	M $\tan\delta - \tan\phi$	ΔT	$\frac{d(\Delta T)}{d\phi}$
δ_n	50°	10° 00'	$8^h35^m\ 9^s.597$	$8^h33^m53^s.388$	$+1^m16^s.209$	0.352654	$-60^s.000$	$0^s.00038/1''$
δ_s	5	35 00	8 40 45.454	8 40 20.000	+ 25.454	-0.751611		
δ_n	55	15 00	8 45 17.430	8 43 50.356	+1 27.074	0.58.7048	-60.000	0.00041
δ_s	15	25 00	8 49 44.155	8 49 10.407	+0 33.748	-0.571150		
δ_s	20	20 00	8 53 49.442	8 53 11.280	0 38.162	-0.475129	-60.000	0.00024
δ_n	45	5 00	8 57 33.195	8 56 25.800	1 07.395	+0.160900		
δ_s	-10	50 00	9 03 34.937	9 03 21.609	0 13.328	-1.015427	-60.000	0.00038
δ_n	47	7 00	9 05 25.033	9 04 14.311	1 10.722	+0.233269		
δ_n	60	20 00	9 09 58.030	9 08 03.306	1 54.724	+0.892951		
δ_s	-30	70 00	9 14 43.602	9 15 10.408	- 26.806	-1.416450	-60.000	0.00059

Both stars on the same quadrant

δ_n	20	20 00	8 53 49.442	8 53 11.280	0 38.162	-0.475129	-60.000	0.00038
δ_s	-10	50 00	9 03 34.937	9 03 21.609	0 13.328	-1.015427		

Latitude: $\phi = 40°$

Azimuth error: $\delta A = 20'$

$$\frac{d(\Delta T)}{d\phi} = \frac{(\beta_s - \beta_n)\sin 1''\,(1 + \tan^2\phi)}{\tan\delta_n - \tan\delta_s}$$

UNCERTAINTY IN EITHER STAR TRANSIT TIMES
or
RIGHT ASCENSION

Consider observations of stars that transit a vertical plane close to the meridian.

The uncertainty in the star transit time is partially due to the inability of the observer to track the star's image motion correctly, the effect of refraction and to a lesser degree, an error in the star's right ascension.

The errors in the transit times or in right ascensions can be detected by using the following equation of condition:

$$\beta_1 (\tan \delta_3 - \tan \delta_2) + \beta_2 (\tan \delta_1 - \tan \delta_3) + \beta_3 (\tan \delta_2 - \tan \delta_1) = 0 \quad (73)$$

where the β's have the meaning:

$$\beta = \theta + i \frac{\cos(\phi - \delta)}{\cos \phi} - \alpha + C_0$$

and

θ = sidereal transit time

i = inclination error

α = right ascension

C_0 is a constant, arbitrarily chosen in order to have small values of β. Equation (73) is to be used in order that the stars are selected in the order of increasing or decreasing declinations

$$\delta_1 > \delta_2 > \delta_3$$

no matter in what order the stars were observed.

In view of the small size of the β's, only a few digits are used for the tan δ's values. A set of independent equations of condition are to be considered. They should be exactly satisfied, but in general they yield discrepancies. Designating by Wi the discrepancies that result in applying

equation (73) and by $d\,\beta_i$ the corrections to the values β_i, each equation of condition has the form

$$A_i \cdot d\beta_i + B_j \cdot d\beta_j + C_k \cdot d\beta_k + W = 0 \qquad (74)$$

There will be n equations with r unknowns. The numbers of unknown r's are greater than n, so the computations of the corrections $d\beta$'s are carried out according to the method of least squares, conditional observations. After the adjustment has been carried out the correct values for the β's should be

$$\beta_1 + d\,\beta_1\ ;\ \beta_2 + d\beta_2\ ;\ \ldots\ldots \text{ and so on.}$$

Let $\delta\theta$ be the error committed in the star's transit time, and let $\delta\alpha$ be the error in the right ascension, then

$$d\beta = \delta\theta - \delta\alpha \qquad (75)$$

We cannot define independently $\delta\theta$ and $\delta\alpha$ Nevertheless $(\beta + d\beta)_i$ are the values to be used in equation (52) to obtain a unique and the best value of ΔT. Any two stars will give the same ΔT value.

If we assume there is no error in the star's right ascension, then follows:

$$d\beta_i = \delta\theta_i \qquad (76)$$

which are the values to be considered using equation (74). Choosing three stars under the condition shown in (79), the discrepancy W is obtained from equation (73). The resulting equation of condition is

$$\delta\theta_1\ (t_3-t_2) + \delta\theta_2(t_1-t_3) + \delta\theta_3(t_2-t_1) + \omega = 0 \qquad (77)$$

where the t's are introduced for abbreviation:

$$t_1 = \tan\,\delta_1$$

$$t_2 = \tan\,\delta_2$$

$$t_3 = \tan\,\delta_3$$

The weight coefficient of $\delta\theta_i$ in equation (77), is $\cos\,\delta_i$.

Consider observations of six stars selected that three are north of zenith and the other three south of the zenith. Symbolize the north stars by N·1, N·2 and N·3; and by S·1, S·2 and S·3 for the south stars selected in the order of having

$$\delta_1 > \delta_2 > \delta_3 \tag{78}$$

The observations can be made in the order of increasing right ascension but chosen the δ's of any three stars in the order shown by (78). The three stars are selected in the order shown in Table 1 which is self-explanatory.

Table 1. Selection of Stars.

1	2	3
N · 1	N · 2	S · 1
N · 1	N · 3	S · 2
N · 2	S · 2	S · 3
N · 3	S · 1	S · 3

These are the four stars selected to fit equation (77).

The computation of the corrections $\delta\theta$ is carried out according to the method of least squares, conditional observations. After the adjustment has been carried out, the correct values for the β's should be

$$\beta_1 + \delta\theta_1;\ \beta_2 + \delta\theta_2;\ \text{and so on}$$

In equation (73) it was assumed that no collination exists. This condition prevails when each star is tracked before and after crossing the vertical plane. This condition may not be realized in practice as if the stars are observed half with instrument clamp west and the other half tracked with instrument clamp east.

In this condition the $d\beta_i$ values to be considered are:

$$d\beta_i = \delta\theta_i \pm c \cdot \sec \delta_i \tag{79}$$

where c represents the collination error. The positive sign is for instrument clamp west and negative for instrument clamp east.

The equation of condition is, for this type of observation:

$$\begin{aligned} &\delta\theta_1 (t_3-t_2) + \delta\theta_2(t_1-t_3) + \delta\theta_3(t_2-t_1) \\ &\pm c\ t_3(\sec \delta_1 - \sec \delta_2) + t_2(\sec \delta_3 - \sec \delta_1) + t_3(\sec \delta_2 - \sec \delta_3) + W = 0 \end{aligned} \tag{80}$$

Forming the equations in the way shown on Table 1 and after the adjustment the corrections to the β's are:

Clamp West	Clamp East	
$\beta_1 + \delta\theta_1 + c \cdot \sec \delta_1$	$\beta_1 + \delta\theta_1 - c \cdot \sec \delta_1$	(81)
$\beta_2 + \delta\theta_2 + c \cdot \sec \delta_2$	$\beta_2 + \delta\theta_2 - c \cdot \sec \delta_2$	

and so on.

COLLIMATION CONSTANT AND ITS EVALUATION FROM A STAR OBSERVATION

The line of sight of the instrument is in general not perpendicular to the horizontal axis of the instrument, but forms an angle with it of 90°+c where c is a quantity called collimation. Consider its evaluation.

A star can be fixed in space by spherical coordinates with respect to two fundamental planes. In determining the azimuth these two fundamental planes are: the horizon and the meridian. The first is defined as normal to the plumb line, and the second is defined by the plumb line and a parallel to the earth's axis of rotation. This plumb line does not, in general, pass through the center of the earth.

The plumb line pierces the celestial sphere at two points: the zenith and the nadir, which lie respectively, directly above and below the observer's point on earth. These two points are the poles of the fundamental circle formed where the horizontal plane cuts the sphere.

Figure 4, shows the celestial sphere with its center at C. The line ZN_0 is the vertical, Z is the zenith and N_0 is the nadir. The great circle NWS, the poles of which are Z and N_0, is the horizontal circle. P_nP_s is the line of poles. The great circle P_nZS is the meridian. These two planes intercept in a line, NS, which represents the North-South direction.

In this paper "azimuth" and "altitude" or "zenith distance" are used as spherical coordinates. A point, A, in space and the line ZN_0 form a plane which cuts the sphere in a great circle of arc, ZAA_0. In this figure A represents a star; A_s its azimuth, and h_s the altitude, A_0A.

Due to the collimation error, the telescope will not describe the great circle of arc ZA_0N_0 but will drscribe one smaller, $Z'A_1$. The separation between these two circles is c, the collimation error, so the star is not

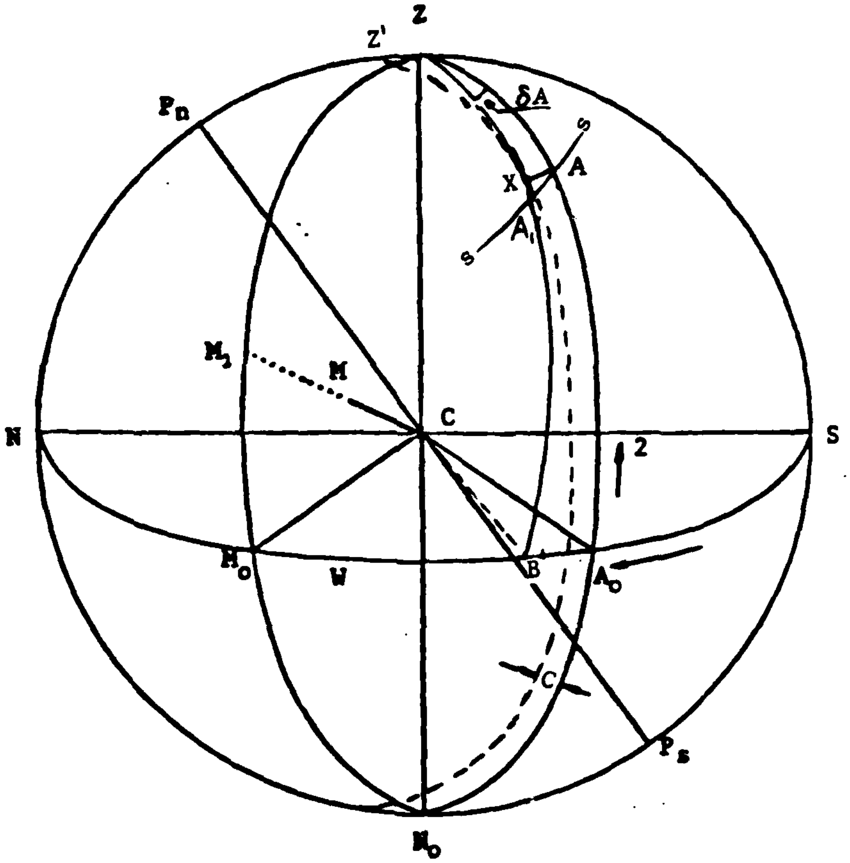

Fig 4. Showing azimuth configuration.

observed at A but at A_1 over the star's parallel ss. The great circle of arc Z and A_1, intercepts the horizontal plane at B. Therefore the angle $BCA_0 = \delta A$ is the correction to be included in the star's azimuth As. To evaluate δA, consider through A a normal to ZB. Since the collimation $c = AX$ is a small arc, we may assume that A A_1X is a plane triangle, right angle at X. In the triangle ZXA we have

$$AX = c = \delta A \sin Z \tag{81}$$

from which the azimuth correction due to collimation error, is

$$\delta A = c/\sin Z \tag{82}$$

Consider the projection onto the horizontal plane SA_0N. The zenith is projected on C. The meridian plane intersects the horizon plane in the line NCS. The azimuth A_s, is measured from the origin at S, south, in the direction of the arrow from 0° to 360° clockwise, so the star's azimuth is

$$A_s = SCA_0$$

This angle cannot be measured directly because the origin S is unknown. To deal with the collimation evaluation we proceed as follows:

Consider a diagram of a theodolite as shown in Fig. 5. Let C represents the center of the horizontal circle. Let NS be the meridian line through C. Let the telescope be pointed to the star A, and let CA' be its horizontal projection. This direction CA' has a reading L_s in the horizontal circle. Let B_0, be the origin of the horizontal scale reading of the theodolite and let B_1 be its symmetrical point. Then the angle formed by the line B_0CB_1 with respect to the meridian line NCS is the azimuth of B_0B_1, that is SCB_0. Let this angle be represented by β, so the star's azimuth A_s, is

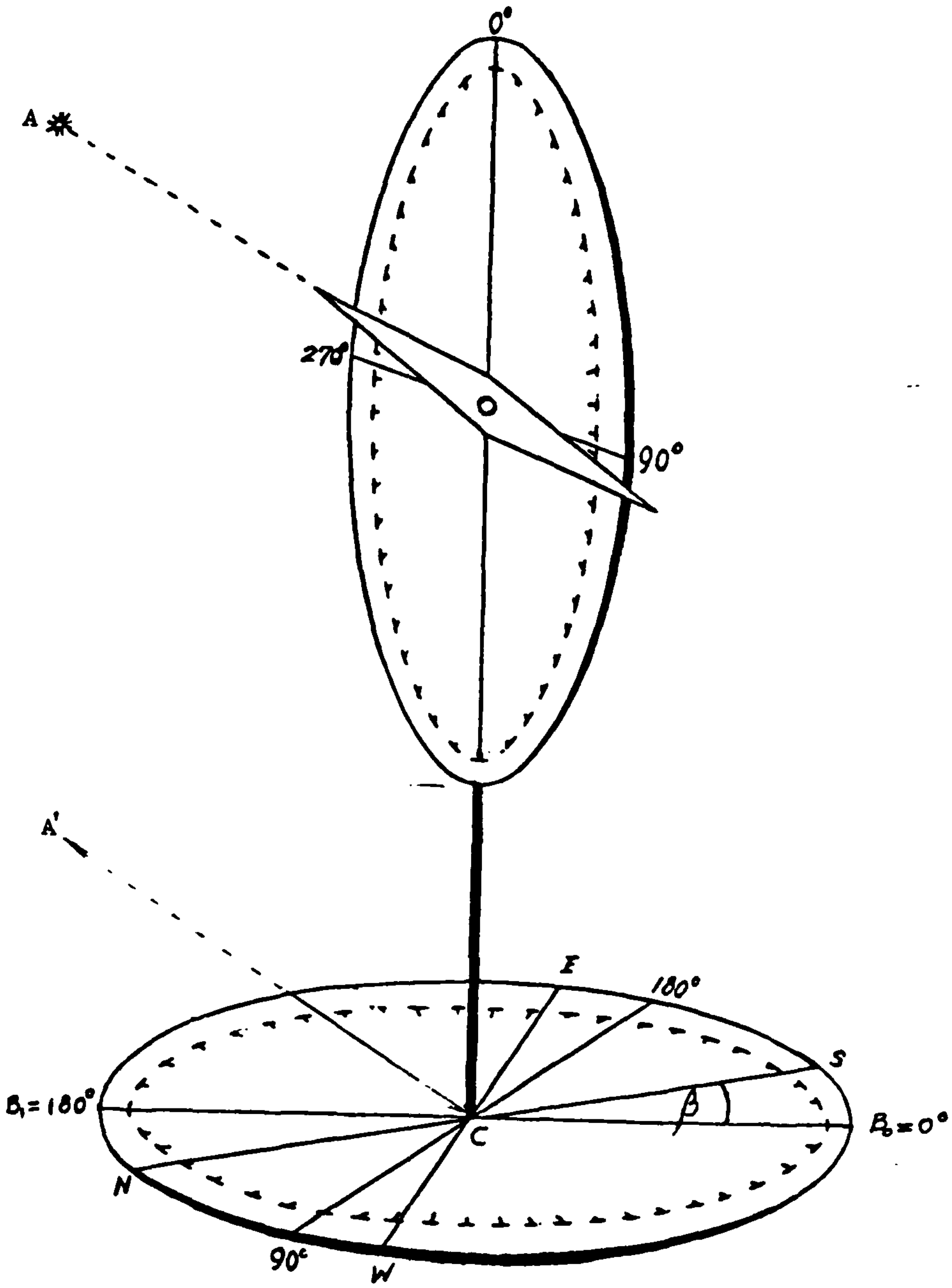

Fig. 5 **Diagram of theodolite.**

$$A_s = L_s + \beta \qquad (83)$$

which represents the value when no collimation exists, but always a small collimation error exists, so we must add to equation (83) the value δA shown on equation (82) therefore, finally, the star's azimuth is

$$A_s = L_s + \beta \pm c \operatorname{cosec} Z \qquad (84)$$

where the plus sign is assumed when the theodolite is in a direct position and negative sign when the instrument is in reverse position.

Consider now how the collimation can be evaluated. It can be determined by selecting any terrestrial object that presents some well defined point, and at a far distance that the stellar focus of the telescope need not be changed to obtain a good definition of the point. From the horizontal readings of the theodolite in a direct and a reverse position the collimation can be obtained. But we consider that this collimation should not be the same when it is derived from a star's observations. We consider that it is more appropriate to obtain the collimation constant by observing a moving object, a star, which is related to the observer's behavior in observing the stars for latitude and longitude determination. Instead of a distant terrestrial point, we substitute it by choosing a circumpolar star. The rate of change of azimuth as function of time is consider to evaluate the collimation constant. The star is observed a number of times in a direct and reverse instrument position. The time when the star image crosses the central vertical wire is recorded as well as the horizontal readings of the theodolite. Let L_i be the horizontal scale readings and T_i be the corresponding times for a direct instrument position and L_j, T_j the values that corresponds to the reverse instrument position, where

$$i, j = 1, 2, 3, \ldots n$$

Let T_o be an arbitrary time chosen between the two sets of observations, and let us reduce all observations, to the time T_o through the Taylor's theorem, expressing all quantities in radiances and omitting the terms higher than the third.

The amount ΔA_i to be added to each L_i to reduce it to the value that would correspond to the time T_o, is

$$\Delta A_i = \frac{\partial A}{\partial t} \sigma_i + \frac{1}{2} \frac{\partial^2 A}{\partial t^2} \sigma_i{}^2 + \frac{1}{6} \frac{\partial^3 A}{\partial t^3} \sigma_i{}^3 \tag{85}$$

where

$$\sigma_i = T_i - T_o$$

and the amount ΔA_j to be added to the readings L_j to reduce them also to the value at T_o, is

$$\Delta A_j = \frac{\partial A}{\partial t} \sigma_j + \frac{1}{2} \frac{\partial^2 A}{\partial t^2} \sigma_j{}^2 + \frac{1}{6} \frac{\partial^3 A}{\partial t^3} \sigma_j{}^3 \tag{86}$$

and

$$\sigma_j = T_j - T_o$$

Then the star's azimuth to the time T_o are

$$A_s = L_i + \beta + \Delta A_i + c \operatorname{cosec} Z_i \qquad \text{Direct}$$

$$A_s = L_j + \beta + 180 + \Delta A_j - c \operatorname{cosec} Z_j \qquad \text{Reverse}$$

Subtracting the first equation from the second we get the evaluatiom of the collimation from

$$c = \frac{L_j + 180 - L_i + \Delta A_j - \Delta A_i}{\text{cosec } Z_i + \text{cosec } Z_j} \tag{87}$$

or by

$$c = \frac{L_j + 180 - L_i + \Delta A_j - \Delta A_i}{\sin Z_i + \sin Z_j} \sin Z_i \sin Z_j \tag{88}$$

The evaluation of the coefficients, $\frac{\partial A}{\partial t}$, $\frac{\partial^2 A}{\partial t^2}$ and $\frac{\partial^3 A}{\partial t^3}$ are shown in Appendix A. Let us apply this method to the reduction of the observations shown on Table III. The first column is the observing time; the second column comprise the observed zenith already corrected for index error and refraction and the third column are the horizontal reading in relation to the time of column 1.

The corrections according to equations (85) are given in the Table under corrections I, II, and III and the symbol Σ in the subsequent column indicates their sum. The last column shows the azimuth values that correspond to the time T_o. Table III shows the values that correspond to the reverse theodolite position. The collimation was computed with the mean values from the last columns of Tables II and III. The process of computing the collimation error and the corresponding sign for the direct and reverse instrument positions is shown at the bottom of Table III.

PLACE: $\lambda = 3^h\ 36^m,\ 26^s.7$ WEST
$\varphi = -26^\circ\ 37'\ 22''$ S

DATE: Feb. 22-1955
STAR: α PICTORIS

OBSERVER: A.A.B.
INSTRUMENT: THEODOLITE ZEISS 1'

TABLE II
TELESCOPE DIRECT POSITION

1	2	3	Corrections			Σ	Reduced Azimuth
Sidereal Time	Zenith Distance	Horizontal Angle	I	II	III	I+II+III	$L_i - \Sigma +$ c cosec Z
$11^h\ 49^m 06^s.4$	60° 02' 15"	31° 46' 33"	+ 5' 06".50	- 1' 42" 10	- 1".28	+ 3' 23".1	31 43' 09".9 + 1.154 c
50 36.6	60 12 54	46 16	4 23. 33	1 15. 37	.81	3 07.2	08.9 + 1.152
51 05.2	60 16 15	46 10	4 09. 64	1 07. 73	.69	3 01.2	08.8 + 1.152
51 34.6	60 19 41	46 04	3 55. 57	1 00. 31	.58	2 54.7	09.3 + 1.151
52 16.0	60 24 35	45 52	3 35. 76	0 50. 59	.45	2 44.7	07.3 + 1.150
52 51.9	60 28 49	45 42	3 18. 56	0 42. 86	.35	2 35.4	06.6 + 1.150
53 20.2	60 32 10	45 35	3 05. 03	0 37. 21	.28	2 27.5	07.5 + 1.149
54 24.5	60 39 43	45 15	2 34. 25	0 25. 86	.16	2 08.2	06.8 + 1.147
11 55 08.6	60 44 59	31 44 59	+ 2 13. 63	- 0 19. 41	- 0.11	+ 1 54.1	31 43 07.9 + 1.146 c

Mean: $31^\circ 43'$ 08".11 + 1.150 c

TABLE III

TELESCOPE REVERSE POSITION

1	2	3	Corrections			Σ	Reduced Azimuth
Sideral Time	Zenith Distance	Horizontal Angle	I	II	III	I+II+III	$L_i - \Sigma -$ c Cosec Z
$12^h\ 04^m 26^s.0$	298° 09' 20"	211° 40' 54"	+ 2' 13".63	- 0' 19".41	+ 0".11	- 2' 32".9	211° 43' 26".9 - 1.134 c
05 42 8	298 00 20	40 06	2 50.38	0 31.55	.22	3 21.7	27.7 - 1.133
06 37 6	297 53 55	39 28	3 16.61	0 42.01	.34	3 58.3	26.3 - 1.132
07 23.7	48 30	38 56	3 38.67	0 51.97	.47	4 30.2	26.2 - 1.131
07 58.9	44 23	38 33	3 55.52	1 00.29	.58	4 55.2	28.2 - 1.130
08 28.8	40 52	38 11	4 09.83	1 07.84	.69	5 17.0	28.0 - 1.129
09 08.6	36 12	37 41	4 28.88	1 18.58	.86	5 46.6	27.6 - 1.128
09 56.5	30 35	37 06	4 51.81	1 32.55	1.11	6 23.2	29.2 - 1.127
10 35.8	297 26 00	211 36 37	- 5 10.61	- 1 44.86	+ 1.13	- 6. 54.1	211 43 28.1 - 1.127 c

Mean: 211° 43' 27".58 - 1.130 c

Collimation error: $A = L_1 + B + C \operatorname{cosec} Z$, Direct

$A = L_2 + B - 180^\circ - c \operatorname{cosec} Z_2$, Reverse

$0 = L_1 + B - 180^\circ + c (\operatorname{cosec} Z_1 + \operatorname{cosec} Z_2)$

$0 = -19''47 + 2.280\ c$ $\quad c = \pm 8''.54$ +Direct
-Reverse

CONCLUSIONS

In this report the procedures for time (longitude) determination by means of transit times of two or more stars through the same vertical plane, fixed close to 20 arc minutes with respect to the meridian and of the method of latitude determination by means of transit times of two stars over a vertical plane fixed close to the prime vertical, are given. Both methods are independent of the instrument azimuth orientation and the two stars forming a pair can be arbitrarily chosen with respect to declination or zenith distances. Provided there is a large difference in the zenith distances of a star pair, both stars can be observed in the same quadrant. The latitude is obtained independently of the clock correction or longitude. Short periods of clear sky observations may then be utilized. When several star pairs are observed an adjustment procedure allows one to correct observational transit times or star positions.

APPENDIX A

In order to compute the ΔA_i and ΔA_j shown in equations (85) and (86), the Taylor's coefficient has been derived in order to have them as function of

δ = star's declination

ϕ_o = approximate value of latitude

z = star's zenith distance that should correspond to the reference time $T=T_o$

The derivation of formulas giving the coefficients in terms of these quantities, are given below:

Differentiate the formulas

$$\begin{aligned} \sin z \sin A &= \cos\phi \sin t \\ \sin z \cos A &= -\cos\phi \sin\delta + \sin\phi \cos\delta \cos t \end{aligned} \tag{A-1}$$

with respect to t, we have

$$\begin{aligned} \cos z \sin A \frac{dZ}{dt} + \sin z \cos A \frac{dA}{dt} &= \cos\varphi \cos t \\ \cos z \cos A \frac{dz}{dt} - \sin z \sin A \frac{dA}{dt} &= -\sin\delta \sin\phi \sin t \end{aligned} \tag{A-2}$$

from which we obtain

$$\sin z \frac{dA}{dt} = \cos\delta(\cos t \cos A + \sin t \sin A \sin\phi) \tag{A-3}$$

$$\cos z \frac{dz}{dt} = \cos\delta(\cos t \sin A - \sin t \cos A \sin\phi) \tag{A-4}$$

The terms in parenthesis in equation (A-3) is cos P and similarly in (A-4) is sin P cos z, so it results

$$\sin z \frac{dA}{dt} = \cos\delta \cos P \tag{A-5}$$

$$\frac{dZ}{dt} = \cos \delta \sin P \tag{A-6}$$

From

$$\sin \phi = \cos z \sin \delta + \sin z \cos \delta \cos P \tag{A-7}$$

(A-5) becomes

$$\sin z \frac{dA}{dt} = \frac{\sin \phi - \cos s \sin \delta}{\sin z} \tag{A-8}$$

differentiating (A-7) with respect to t, we get

$$\sin z \frac{d^2A}{dt^2} = (\sin \delta - 2 \frac{dA}{dt} \cos z) \frac{dZ}{dt} \tag{A-9}$$

Differentiating again and after some simplifications, we obtain

$$\frac{d^3A}{dt^3} = \frac{d^2z}{dt^2} (\frac{\sin \phi}{\sin z} - 2 \cot z \frac{dA}{dt}) + 2(\frac{dZ}{dt})^2 \frac{dA}{dt} - 3 \cot z \frac{dZ}{dt} \frac{d^2A}{dt^2} \tag{A-10}$$

From (A-7)

$$\frac{dA}{dt} = \frac{\sin \phi - \cos z \sin \delta}{\sin^2 z} \tag{A-11}$$

and $\frac{dZ}{dt}$ is computed from

$$\frac{dz}{dt} = \pm \sqrt{\cos^2 \delta - \left(\sin z \frac{dA}{dt}\right)^2} \quad \begin{matrix} + \text{ star west} \\ - \text{ star east} \end{matrix} \tag{A-12}$$

It remains to know $\frac{d^2z}{dt^2}$

From (A-6) we have

$$\frac{d^2z}{dt^2} = \cos \delta \cos P \frac{dP}{dt} \tag{A-13}$$

From the equation (A-7) we get

$$0 = \frac{dZ}{dt} (-\sin z \sin \delta + \cos z \cos \delta \cos P)$$
$$- \sin z \cos \delta \sin P \frac{dP}{dt}$$

The term in parenthesis is $\cos \phi \cos A$ and from (A-6) we obtain

$$\frac{dP}{dt} = \frac{\cos \phi \cos A}{\sin z}$$

From which (A-13) becomes

$$\frac{d^2 z}{dt^2} = \frac{\cos \phi \cos A \cos \delta \cos P}{\sin z} \tag{A-14}$$

From (A-7) and

$$\sin \delta = \cos z \sin \phi - \sin z \cos \phi \cos A \tag{A-15}$$

we obtain

$$\frac{d^2 z}{dt^2} = \cos z (\sin^2 \phi + \sin^2 \delta) - \sin \delta \sin \phi (1 + \cos^2 z) \tag{A-16}$$

Then the Taylor's coefficient are

$$\frac{dA}{dt} = \frac{\sin \phi - \cos z \sin \delta}{\sin^2 z} \tag{A-17}$$

$$\frac{d^2 A}{dt^2} = \left(\frac{\sin \delta}{\sin z} - 2 \frac{dA}{dt} \cot z\right) \frac{dZ}{dt} \tag{A-18}$$

The rate $\frac{dZ}{dt}$ is known from equation (A-16) and

the $\frac{d^3 A}{dt^3}$ is computed from equation (A-10) using:

$\frac{d^2 Z}{dt^2}$ from equation (A-16)

$\frac{dA}{dt}$ from equation (A-17)

$\frac{dz}{dt}$ from equation (A-12)

These Taylor's coefficients, A-17, A-18 and A-16 are computed with the star's zenith distance that correspond to the reference time $T=T_o$, after having been corrected for refraction and index error, an approximate value of the observer's latitude and star's declination.

APPENDIX B

REDUCTION OF TRANSIT TIMES OF A STAR OVER SEVERAL WIRES WITH THE TELESCOPE FIXED TO THE CENTER WIRE

The Taylor coefficients to be used in equation (38) herein are derived.

A star's hour angle rate, with respect to azimuth, is related to the parallactic angle P, and zenith distance z, through the equation,

$$\frac{dt}{dA} \cos \delta \cos P = \sin z \qquad \text{(B-1)}$$

Differentiating this equation with respect to A, we have

$$\frac{d^2t}{dA^2} \cos \delta \cos P - \frac{dt}{dA} \cos P \sin P \frac{dP}{dA} = \cos z \frac{dz}{dA} \qquad \text{(B-2)}$$

and a second derivation gives for $\frac{d^3t}{dA^2}$

$$\frac{d^3t}{dA^3} - 2\frac{d^2t}{dA^2} \tan P \frac{dP}{dA} - \frac{dt}{dA} \tan P \frac{d^2z}{dA^2}$$
$$= - \frac{\sin z}{\cos \delta \cos P} \left(\frac{dz}{dA}\right)^2 + \frac{\cos z}{\cos \delta \cos P} \frac{d^2z}{dA^2} \qquad \text{(B-3)}$$

In the prime vertical, A = ± 90°, $\frac{dP}{dt} = 0$, as shown in Equation (2) therefore equation (B-2) and (B-3) becomes,

$$\left(\frac{dt^2}{dA^2}\right)_{A = \pm 90°} = \frac{\cos z}{\cos \delta \cos P} \frac{dz}{dA} \qquad \text{(B-4)}$$

$$\left(\frac{d^3t}{dA^3}\right)_{A = \pm 90} = \frac{dt}{dA} \tan P \frac{d^2P}{dA^2} - \frac{\sin z}{\cos \delta \cos P} \left(\frac{dz}{dA}\right)^2$$
$$+ \frac{\cos z}{\cos \delta \cos P} \frac{d^2z}{dA^2} \qquad \text{(B-5)}$$

The rate $\frac{dz}{dt}$ is obtained from equations (A-5) and (A-6),

$$\frac{dz}{dA} = \tan P \sin z \tag{B-6}$$

Differentiating it with respect to A, we obtain,

$$\frac{d^2z}{dA^2} = (1 + \tan^2 P) \sin z \frac{dp}{dA} + \tan P \cos z \frac{dz}{dA} \tag{B-7}$$

which in the prime vertical, reduces to

$$\frac{d^2z}{dA^2} = \tan P \cos z \frac{dz}{dA} \tag{B-8}$$

Replacing $\frac{dz}{dt}$ from equation (B-6) we get

$$\frac{d^2z}{dA^2} = \tan^2 P \sin z \cos z \tag{B-9}$$

In order to derive $\frac{d^2z}{dA^2}$, we use the equation,

$$\cos \phi \sin A = \cos \delta \sin P \tag{B-10}$$

The first differentiation of this equation gives,

$$\cos \phi \cos A = \cos \delta \cos P \frac{dP}{dA} \tag{B-11}$$

and from a second differentiation, we obtain

$$-\cos \phi \sin A = -\cos \delta \sin P \left(\frac{dP}{dA}\right)^2 + \cos \delta \cos P \frac{d^2P}{dA^2} \tag{B-12}$$

which for $A = \pm 90°$, results

$$\frac{d^2P}{dA^2} = -\tan P \tag{B-13}$$

With equations (B-6), (B-8) and (B-13) the Taylor coefficients were obtained as follows

$$\left(\frac{dt}{dA}\right)_{A = \pm 90°} = \frac{\sin z}{\cos \delta \cos P} \tag{B-14}$$

$$\left(\frac{d^2t}{dA^2}\right)_{A = \pm 90°} = \frac{\cos z \sin z \tan P}{\cos \delta \cos P} \tag{B-15}$$

$$\left(\frac{d^3t}{dA^3}\right)_{A = \pm 90°} = -2\,\frac{\sin^3 z \tan^2 P}{\cos \delta \cos P} \tag{B-16}$$

The parallactic angle P is evaluated as function of the stars' pair transit times over the central wire through the equations (23) to (27). As function of P, we obtain the star zenith distance from

$$\tan z = \cos P \cot \delta$$

As can be seen, the Taylor's coefficients can be obtained independent of the station astronomic coordinates.

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